

# Effects of particle plasticity characteristics on local interface stress in particle reinforced composite during uniaxial tension

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**Abstract** For elastoplastic particle reinforced metal matrix composites, failure may originate from interface debonding between the particles and the matrix, both elastoplastic and matrix fracture near the interface. To calculate the stress and strain distribution in these regions, a single reinforcing particle axisymmetric unit cell model is used in this article. The nodes at the interface of the particle and the matrix are tied. The development of interfacial decohesion is not modelled. Finite element modelling is used, to reveal the effects of particle strain hardening rate, yield stress and elastic modulus on the interfacial traction vector (or stress vector), interface deformation and the stress distribution within the unit cell, when the composite is under uniaxial tension. The results show that the stress distribution and the interface deformation are sensitive to the strain hardening rate and the yield stress of the particle. With increasing particle strain hardening rate and yield stress, the interfacial traction vector and internal stress distribution vary in larger ranges, the maximum interfacial traction vector and the maximum internal stress both increase, while the interface deformation decreases. In contrast, the particle elastic modulus has little effect on the interfacial traction vector, internal stress and interface deformation.

## Introduction

Particle reinforced metal matrix composites have high specific strength, specific modulus and wear resistance and high temperature capability. They have received wide attention from aeronautical, space, automotive, mechanical and electronic fields. A good understanding of the mechanical behaviour of particle reinforced metal matrix composites will help improve materials design and processing. Researchers started studying these since 1980s [1–6]. Fracture of particle reinforced metal matrix composites mainly appears as debonding between particles and the matrix, ductile fracture in the matrix near the interface and fracture of reinforcing particles. Therefore, it is very important to study the mechanical behaviour in the proximity of the particles. During studying the local mechanical behaviour of this type of composites, there are difficulties in experimental design and mathematical complexity in theoretical derivation. For these reasons, computer simulation using the finite element method is widely used.

In computer simulation of these materials, an axisymmetric unit cell model containing a single reinforcing particle is often used. This model assumes that the unit cell contains only one reinforcing particle, in the centre of the symmetrical axis of the unit cell. According to periodical conditions, the unit cell should have a hexagonal shape. In practice, however, for simplifying analysis, the hexagonal shape is often reduced to cylinder. The particle shapes are usually approximated using simple geometrical shapes, for examples, spherical, spheroidal and cylindrical. Because this model can be simplified to two-dimensional, it uses a small number of finite elements, facilitating fast computation. It has attracted wide adoption [1, 7].

In the past, when using computer simulation with the finite element method, usually, the reinforcing particles are

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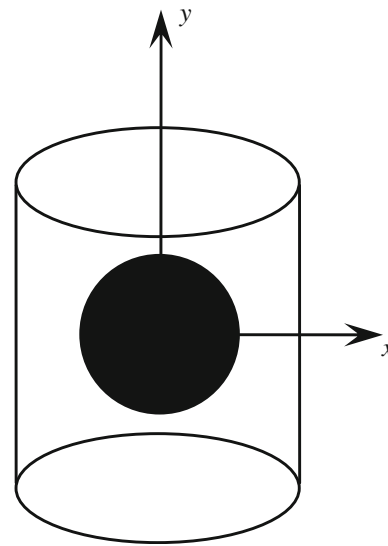
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assumed to the linear elastic. There is a general perception that adding elastic and hard particles in composites strengthens the material [8, 9], while adding ductile particles toughens the material [10]. There is much work on toughening using rubber particles [11]. In practice, different material properties must be considered together. There is soft adhesion of particles on matrix in elastoplastic particle reinforced metal matrix composites, which can improve the local stress distribution in the material. This has attracted attention.  $YAl_2$  particle reinforced Mg–Li matrix composite was developed in [12], with superior properties. However, there has not been literature report on numerical simulation of local stresses in elastoplastic particle reinforced metal matrix composites. This article will carry out such simulation using the single reinforcing particle axisymmetrical unit cell model. Considering the local damage mode of the material, the nodes at the interface of the particle and the matrix are tied. Assuming that reinforcing particles are elastoplastic, we will study the effects of particle strain hardening rate, yield strength and elastic modulus on the mechanical behaviour within the unit cell.

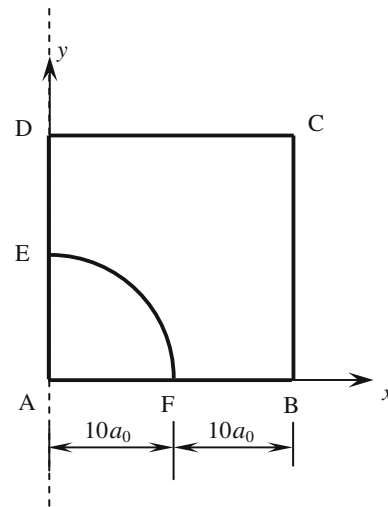
**The computer model and material parameters**

The axisymmetric unit cell model containing a single reinforcing particle is shown in Fig. 1. Its geometrical configuration is shown in Fig. 2. The particle is spherical. Figure 2 shows a quarter of the unit cell model, where AFE is part of the particle and FBCDE is part of the matrix. The radius of the spherical particle is  $10a_0$ . The matrix has a cylindrical shape, and its radius is  $20a_0$ . The height of the cylinder is  $40a_0$ , which can be deduced from Fig. 2. Correspondingly, the volume fraction of particle  $V_f$  is 1/12, i.e. the quotient of the particle volume and the whole cell. During model setting up, due to symmetry, the three-dimensional unit cell was simplified to two-dimensional. The deformable two-dimensional solid structure was used, and it is in the symmetrical model space. The symmetrical axis is the AD side. In model meshing, four-node bilinear symmetrical quadrilateral elements were used. The element type is axisymmetric solid.

The nodes at the interface of the particle and the matrix are tied. In this manuscript, the interface is supposed to have zero thickness. The particle is stuck on the matrix, and the nodes at the interface of the particle and the matrix are fixed together. The nodes at the interface of the particle and the matrix are tied. The Young’s modulus of the matrix and the particle is 76 and 460 GPa, respectively. The Poisson’s ratio of the matrix and the particle is 0.33 and 0.25, respectively. The development of interfacial decohesion is not modelled. As shown in Fig. 2, this assumes a thin layer



**Fig. 1** Schematic diagram of axisymmetric unit cell model containing a single reinforcing particle



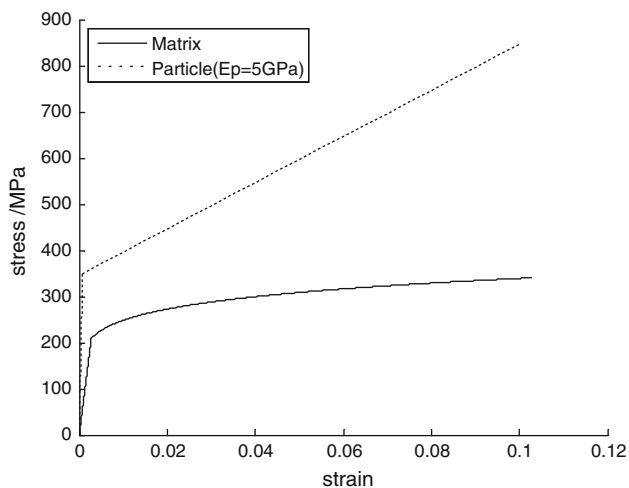
**Fig. 2** Schematic diagram of the geometrical configuration of the unit cell and the interface model

on the interface, which connects the particle and the matrix, ensuring the continuity of the interfacial traction vector between the particle and the matrix and the continuity of displacement.

The stress–strain curves of reinforcing elastoplastic particles and the matrix are shown in Fig. 3. Aluminium alloy is used as the matrix material. A piecewise function used by Xue et al. [13] was adopted to describe its stress–strain relationship:

$$\sigma = \begin{cases} E_{Al}\epsilon & \epsilon \leq \sigma_{s1}/E_{Al} \\ 464\epsilon^{0.136}(\text{MPa}) & \epsilon > \sigma_{s1}/E_{Al} \end{cases} \quad (1)$$

where  $E_{Al}$  is the elastic modulus of the aluminium alloy,  $\sigma_{s1}$  is the yield strength of the aluminium alloy. A linear



**Fig. 3** Stress–strain curves of the elastoplastic particle and the matrix

hardening elastoplastic model is used to describe the particle, with its stress–strain relationship as follows:

$$\sigma = \begin{cases} E\varepsilon & \sigma \leq \sigma_s \\ \sigma_s + E_p \varepsilon_p & \sigma > \sigma_s \end{cases} \quad (2)$$

where  $E$  is the elastic modulus of the particle material,  $\sigma_s$  is its yield strength,  $E_p$  is its strain hardening rate,  $\varepsilon_p$  is its plastic strain, which has a relationship with the total strain  $\varepsilon_p = \varepsilon - \sigma_s/E$ . The linear hardening model is intended to correspond to a sort of strengthen materials, not any specific material, say, YAl<sub>2</sub>, and the matrix material is a kind of Al alloy.

The bulk particle reinforced composite is made up with many such small unit cells, packing repeatedly [1]. When the bulk material is under tension in the  $y$ -direction, for each unit, it may be assumed that there is no stress in the  $x$ -direction along the BC side, and the BC and DC sides remain straight (Fig. 2). In order to achieve these conditions, after applying a displacement of  $2a_0$  of DC in  $y$ -direction, a trial displacement of  $x_0 a_0$  is applied on the BC side along the  $x$ -direction, to enable the following condition along the BC side:

$$\sum_{i=1}^n \sigma_{11}^i = 0 \quad (3)$$

where  $n$  is the node number along the BC side, and the superscript  $i$  refers to the  $i$ th node. If the applied trial displacement cannot make the average stress in the  $x$ -direction along the BC side zero, i.e. Eq. 3 does not stand; a new trial value for  $x_0$  is used, until Eq. 3 is satisfied. This is a gradually approaching process. When Eq. 3 is satisfied, the unit cell state is properly conditioned, to represent a part in the bulk material under uniaxial tension in the  $y$ -direction.

The material is composed of cells periodically distributed, and under the loading the deformation of the cell surfaces is uniform. The upper surface DC maintains a state of plane, which is easy to comprehend and realise through the displacement boundary conditions in the software. BC maintains a uniformly deformed curved surface around the same axis AD. All of the above determined the loading method.

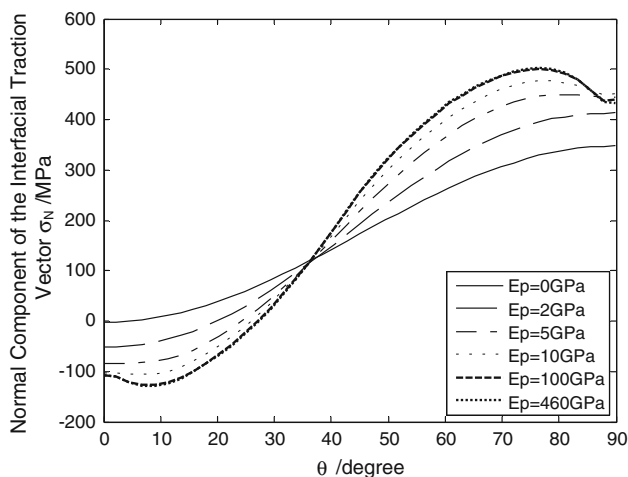
In the simulation, the standard Abaqus (version 6.7) FE package software was used. The boundary conditions at faces AB and AD can be deduced from Figs. 1 and 2, i.e. AD is the axial symmetry axis and AB is an axis of symmetry of the plane ABCD. The unit of ABCD represent a circle of 360°, so the boundary condition of AB and AD is lateral symmetry and axial symmetry, respectively.

Alternatively, with displacement-based finite element algorithms the appropriate radial displacements of axisymmetric cells have usually been obtained by designating one node on face BC as a master node and slaving the radial displacements of all other nodes on this face (the “slave nodes”) to this degree of freedom. Such an approach, on the one hand, does not require iterating (making it more efficient) and, on the other hand, does not explicitly require the nodal stress values at model surfaces, which tend to be inaccurate in displacement-based FE models. The boundary conditions used by the authors may therefore be considered unconventional.

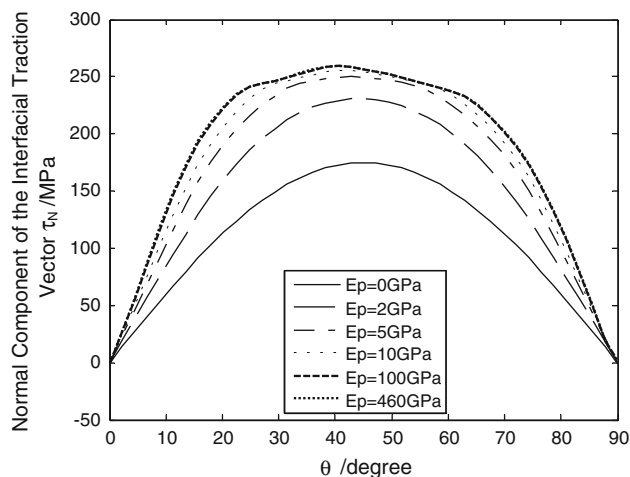
The figures in the following (“The effect of particle strain hardening rate on the local stress at the interface”, “The effect of particle yield strength on the local stress at the interface” and “The effect of particle elastic modulus on the local stress at the interface” sections) will show the effects of particle parameters. Among these, the first group of five figures (“The effect of particle strain hardening rate on the local stress at the interface” section) shows the effect of strain hardening rate of the particle on the interfacial traction vector and particle deformation. The second group of five figures (“The effect of particle yield strength on the local stress at the interface” section) shows the effect of particle yield strength and the third group of five figures (“The effect of particle elastic modulus on the local stress at the interface” section) shows the effect of particle elastic modulus. The material used in the second and the third groups (“The effect of particle yield strength on the local stress at the interface” and “The effect of particle elastic modulus on the local stress at the interface” sections) is ideal plastic. In all these figures, the value of the uniaxial macroscopic displacement of the DC side is  $2a_0$ . The direction of the profiles is with respect to the direction of  $E \rightarrow F$ .  $\theta = 0$  corresponds to point  $E$  and  $\theta = 90^\circ$  corresponds to point  $F$ .

**The effect of particle strain hardening rate on the local stress at the interface**

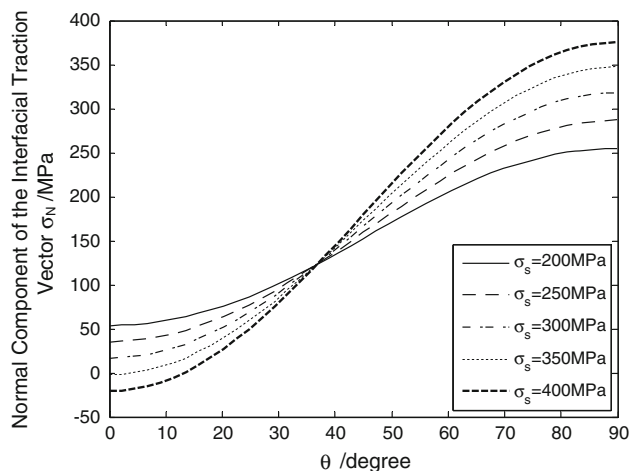
It can be seen from Fig. 4 that, with increasing strain hardening rate of the particle, the variation of the normal component of the interfacial traction vector increases, and the maximum normal component of the interfacial traction vector increases. For an  $E_p$  of 460 GPa, i.e. when the particle is an elastic particle, the maximum tensile stress is 502 MPa, the maximum compressive stress is 128 MPa, which are at  $\theta = 76.5^\circ$  and  $\theta = 9^\circ$ , respectively. When  $\theta$  is smaller than  $9^\circ$ , with increasing  $\theta$ , the compressive stress increases. When  $\theta$  is in the range of  $9^\circ$  and  $76.5^\circ$ , with increasing  $\theta$ ,  $\sigma_N$  changes from compressive to tensile, and increases monotonously. When  $\theta$  is greater than  $76.5^\circ$ ,  $\sigma_N$  reduces gradually. For an  $E_p$  of zero, i.e. when the particle is ideal plastic, the maximum tensile stress is at  $\theta = 90^\circ$ , about 348 MPa, and the minimum stress is zero, at  $\theta = 0^\circ$ . When  $E_p$  changes from 0 to 460 GPa, the magnitude of the normal component of the interfacial traction vector  $\sigma_N$  becomes less sensitive to  $E_p$  when  $E_p$  is large. When  $E_p$  is larger than 100 GPa, there is no obvious influence of the change in  $E_p$  on  $\sigma_N$ . In general, the debonding between the particle and the matrix is mainly determined by the normal component of the interfacial traction vector between the particle and the matrix. From the above analysis, it can be inferred that, compared with elastic particles, plastic particles have a lower demand on the interfacial traction vector between the particle and the matrix. From Fig. 5, it can be seen that, with increasing strain hardening rate of the particle, the tangential component of the interfacial traction vector increases. When  $E_p$  is 460 GPa, i.e. elastic particle, the maximum value of the interfacial tangential stress occurs at  $\theta = 40.5^\circ$ , about 259 MPa. When  $\theta$  is smaller than  $40.5^\circ$ , with increasing  $\theta$ ,  $\tau_N$  increases. When  $\theta$



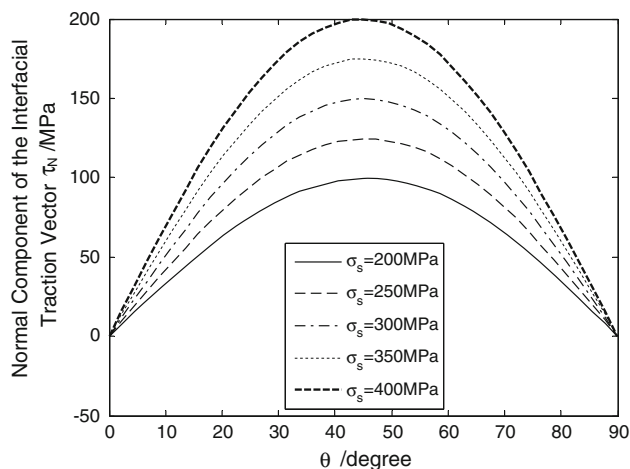
**Fig. 4** The relationship between strain hardening rate  $E_p$  and the normal component of the interfacial traction vector



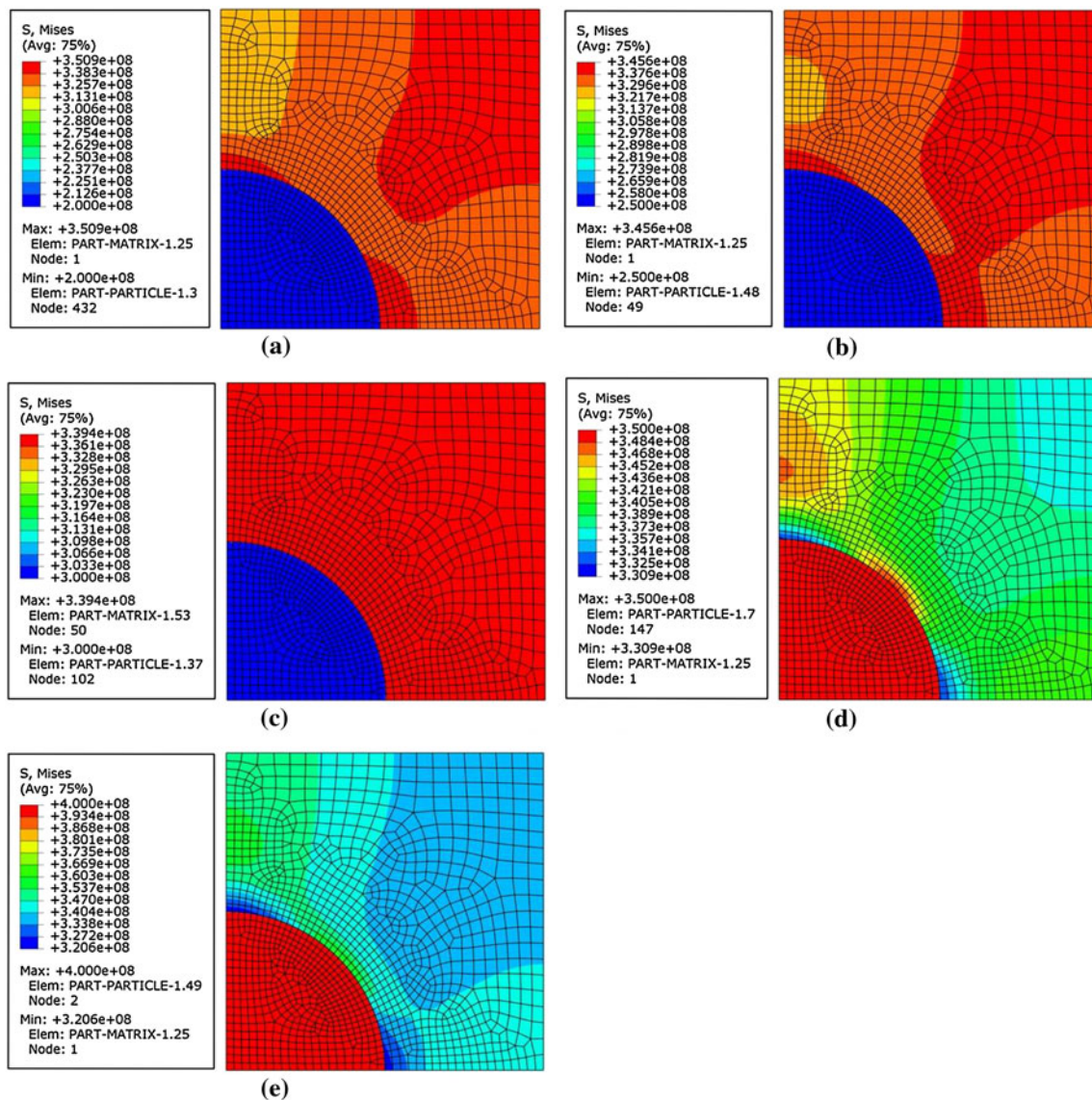
**Fig. 5** The relationship between strain hardening rate  $E_p$  and the tangential component of the interfacial traction vector



**Fig. 6** The relationship between yield strength  $\sigma_s$  and the normal component of the interfacial traction vector



**Fig. 7** The relationship between yield strength  $\sigma_s$  and the tangential component of the interfacial traction vector



**Fig. 8** Contour diagrams of Von Mises stress for different yield strength  $\sigma_s$ . **a**  $\sigma_s = 200$  MPa, **b**  $\sigma_s = 250$  MPa, **c**  $\sigma_s = 300$  MPa, **d**  $\sigma_s = 350$  MPa and **e**  $\sigma_s = 400$  MPa

is greater than  $40.5^\circ$ , with increasing  $\theta$ ,  $\tau_N$  decreases.  $\tau_N$  is zero at  $\theta = 0^\circ$  and  $90^\circ$ . For zero  $E_p$ , i.e. ideal plastic particle, the maximum  $\tau_N$  is about 175 MPa. For different  $E_p$  values, the variation in  $\tau_N$  is 84 MPa. Therefore, the particle material being plastic, instead of elastic, also has a lower demand on the shear resistance of the interface, under uniaxial tension. The shear stress is more sensitive to  $E_p$  for smaller  $E_p$ . When  $E_p$  is greater than 100 MPa,  $E_p$  has little effect on  $\tau_N$ .

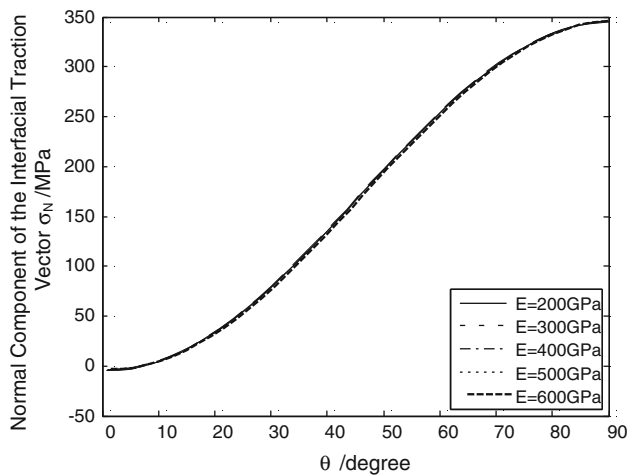
It should be noted that the boundary condition used is displacement, and the applied stress is not a uniform one, so normalised stress values could not be obtained. The complex condition is due to the cell being periodically distributed part of the material.

The yielding amount can be qualitatively deduced from the figures.

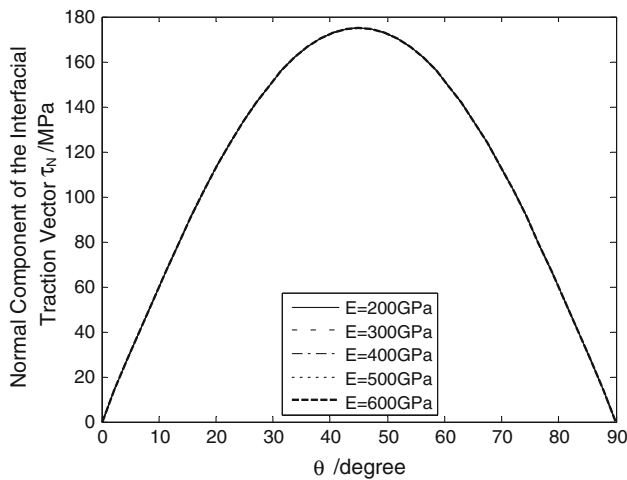
### The effect of particle yield strength on the local stress at the interface

The yield strength  $\sigma_s$  of the particle also influences interaction between the particle and the matrix at the interface. Figures 6 and 7 show the distributions of normal and tangential components of the interfacial traction vector.

As can be seen in Fig. 6, increasing  $\sigma_s$  leads to wider ranges of variation of the normal component of the interfacial traction vector ( $\sigma_N$ ). When  $\sigma_s$  is 400 MPa, the lower limit



**Fig. 9** The relationship between the particle’s elastic modulus  $E$  and the normal component of the interfacial traction vector



**Fig. 10** The relationship between the particle’s elastic modulus  $E$  and the tangential component of the interfacial traction vector

of  $\sigma_N$  at the interface is compressive,  $-20$  MPa, while the upper limit is tensile,  $376$  MPa. When  $\sigma_s$  is  $200$  MPa, however, the lower limit of  $\sigma_N$  at the interface is tensile,  $53.8$  MPa, while the upper limit is tensile,  $255$  MPa. In addition, it should be noted that, when  $\sigma_s$  is  $200$ – $300$  MPa, the normal component of the interfacial traction vector is always tensile, but when  $\sigma_s$  is  $350$ – $400$  MPa, the normal component of the interfacial traction vector changes from compressive at small  $\theta$  to tensile as  $\theta$  increases. This is because, when  $\sigma_s$  is  $\leq 300$  MPa, the Von Mises stress in the matrix is larger than the Von Mises stress in the particle, clearly shown in Fig. 8a–c. In addition, when  $\sigma_s$  is  $350$  MPa, the minimum normal component of the interfacial traction vector is around zero. This does not mean that  $\sigma_s = 350$  MPa is a critical yield stress. This particular value is related to the vertical displacement applied, and thus is largely incidental. Finally, a small  $\sigma_s$  leads to smaller

differences between the upper and the lower limits of the normal component of the interfacial traction vector, showing the advantage of using weaker particles. However, we cannot make the conclusion that, because small  $\sigma_s$  gives uniform interface stress distribution, we should choose a material with the smallest  $\sigma_s$ . In practical applications, small  $\sigma_s$  only indicates that debonding is less likely between the particle and the matrix. It may be detrimental to the overall properties. Therefore, in practice, its combined effect, both microscopically and macroscopically, must be analysed.

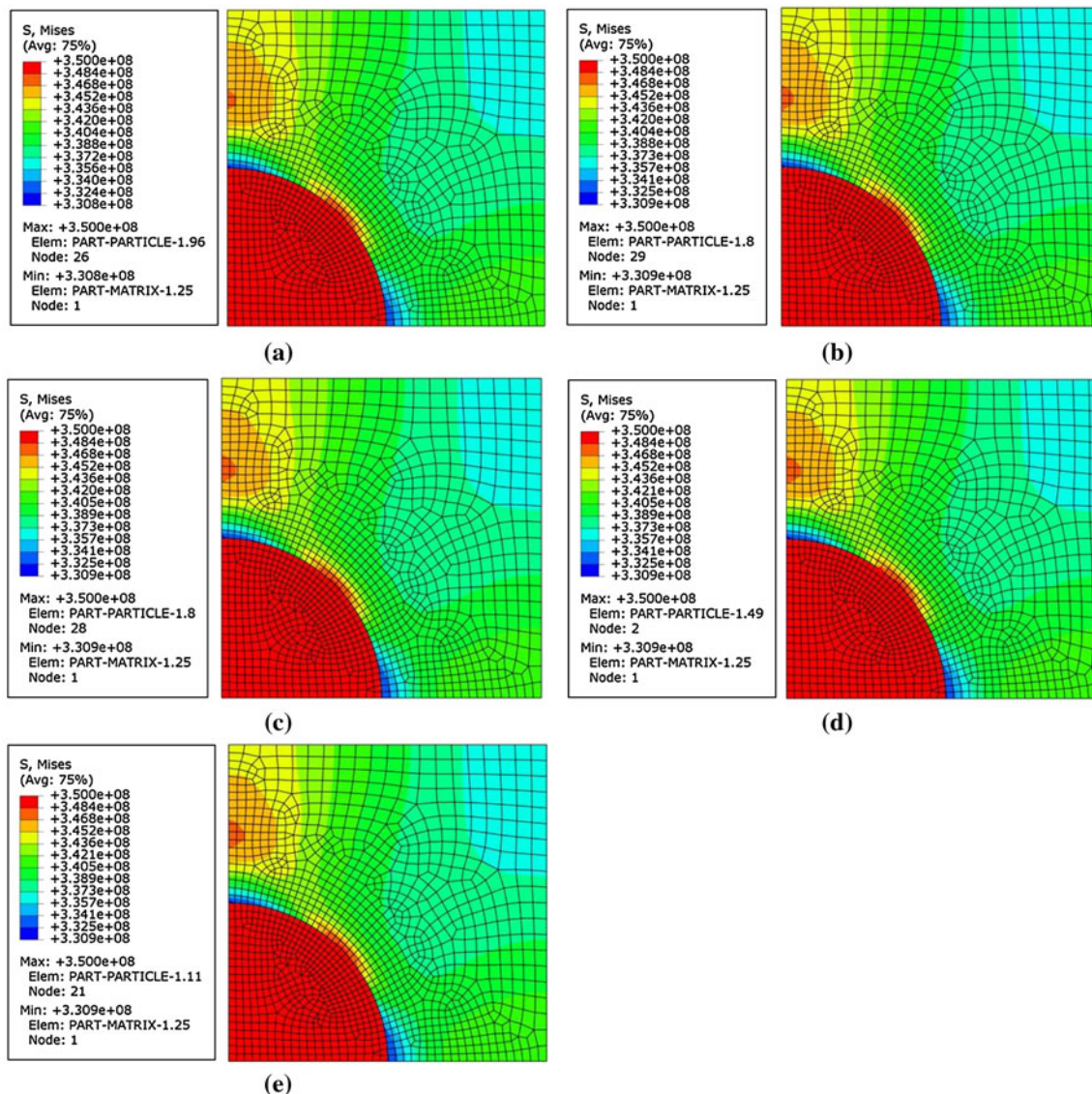
It can be seen in Fig. 7 that, with increasing  $\sigma_s$ , the maximum value of the tangential component of the interfacial traction vector increases, which always occurs at  $\theta = 45^\circ$ . When  $\sigma_s$  is  $400$  MPa,  $\tau_{Nmax}$  is  $200$  MPa. When  $\sigma_s$  is  $200$  MPa, however,  $\tau_{Nmax}$  is  $99$  MPa, showing the advantage of weaker material. Second, along the interface, except at  $F$  point ( $\theta = 0^\circ$ ) and  $E$  point ( $\theta = 90^\circ$ ), increasing  $\sigma_s$  always leads to increasing  $\tau_N$ .  $E$  and  $F$  points are not affected by the yield stress of the particle, and  $\tau_N$  is zero for all values of  $\sigma_s$ . Finally, for any yield stress  $\sigma_s$ ,  $\tau_N$  exhibits a parabola type of variation. With increasing angle  $\theta$ ,  $\tau_N$  first increases and then decreases. In materials design,  $\tau_N$  on the interface at around  $\theta = 45^\circ$  should be given particular attention to.

From the above analysis of Figs. 6 and 7, a lower yield stress results in larger deformation of the particle. This is not really surprising, provided that the applied load is sufficiently high for causing yielding of the particle.

For simplicity, the contours of the Von Mises stress of the composite on only the section of  $x$ – $y$  plane are shown here, in Fig. 8. Under the boundary conditions specified, the particle reached its yield stress in all cases. In the matrix, when  $\sigma_s$  is smaller than  $300$  MPa, higher yield stress results in reductions of the maximum and the minimum Von Mises stresses. When  $\sigma_s$  is larger than  $300$  MPa, higher yield stress results in increases of the maximum and the minimum Von Mises stresses. This value of  $300$  MPa is determined by the boundary conditions in this calculation case, so its exact value is actually incidental. On the other hand, in the matrix, when  $\sigma_s$  is smaller than  $300$  MPa, the Von Mises stress adjacent to the particle interface does not vary significantly. When  $\sigma_s$  is larger than  $300$  MPa, the extreme values of the Von Mises stress on the particle interface differ more with increasing  $\sigma_s$ .

### The effect of particle elastic modulus on the local stress at the interface

Figures 9 and 10 show the influence of particle elastic modulus on the interfacial traction vector. It can be seen that the effect of elastic modulus has a relatively small effect on the material. The distributions of  $\sigma_N$  and  $\tau_N$  each



**Fig. 11** Contour diagrams of Von Mises stress for different particle's elastic modulus  $E$ . **a**  $E = 200$  GPa, **b**  $E = 300$  MPa, **c**  $E = 400$  GPa, **d**  $E = 500$  MPa and **e**  $E = 600$  GPa

overlaps the corresponding curves in Figs. 4 and 5 for  $E_p = 0$ . The calculation results here show that the elastic modulus has minute and negligible effect on these parameters.

In addition, from Fig. 11 showing the contour diagrams on the  $x$ - $y$  section, there is virtually no influence of the particle elastic modulus on the distribution of Von Mises stress. A weak influence is in the exact location of the maximum Von Mises stress. Two regions, in the matrix right above the particle at a distance of about  $5a_0$  and the interface between the particle and the matrix around  $40$ – $50^\circ$ , respectively, have relatively large Von Mises stress. These areas should be given attention to during materials design.

## Conclusion

An axisymmetric unit cell model containing a single reinforcing particle has been used, to simulate particle reinforced metal matrix composites using elastoplastic particles as reinforcements. Finite element modelling has been used, to analyse the effects of particle characteristics on the local stress near the interface in the composite, under uniaxial tension. Based on the simulation and analysis results, the following conclusions can be made:

- (1) The stress distribution and the interface deformation are relatively sensitive to the strain hardening rate and the yield stress. In contrast, the change in the elastic

modulus has relatively little effect on the interface adhesion stress, internal stress and the interface deformation.

- (2) Larger strain hardening rate and yield stress lead to larger variations in the interface adhesion stress and internal stress, resulting in larger maxima of these stresses. However, the interface deformation decreases.

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